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Abstracts

Trigonometric Convexity for the Multidimensional Indicator after Ivanov

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The concept of indicator is well-known for analytic functions in one complex variable. Multidimensional indicator after Ivanov is a generalization of that concept for analytic functions in several complex variables. We state the trigonometric convexity for n-dimensional indicator after Ivanov [1].

Defination 1. Denote by $Exp\,(\alpha_1, \ldots, \alpha_n)$ the class of functions f that are analytic and of finite exponential type in $\Delta_{\alpha_1} \times \cdots \times \Delta_{\alpha_n}$,

$$\text{where } \; \Delta_{\alpha_j} = \{z_j \in \mathbf{C} \setminus \{0\} \colon \; |\arg(z_j)| < \alpha_j < \pi/2\}\,.$$

Defination 2. Namely, Ivanov introduced the following set:

$$T_f(\vec{\theta}) = \{\vec{\nu} \in \mathbf{R}^n : \ln|f(\vec{r}e^{i\vec{\theta}})| \le \nu_1 r_1 + ... + \nu_n r_n + C_{\vec{\nu},\vec{\theta}}, \text{ for all } \vec{r} \in \mathbf{R}^n_+\},$$

here $\vec{r}e^{i\vec{\theta}}$ is the vector $(r_1 e^{i\theta_1}, ..., r_n e^{i\theta_n})$. The set $T_f(\vec{\theta})$ implicitly reflects the notion of an indicator of an entire function.

Theorem *Let a function $f \in Exp\,(\alpha_1, \ldots, \alpha_n)$ and the numbers A_1^+, A_1^- $\ldots$, A_n^+, A_n^- satisfy*

$$\left(A_1^{l_1}, \ldots, A_n^{l_n}\right) \in \overline{T}_f\,(l_1\alpha_1, \ldots, l_n\alpha_n)\,,$$

where $l_j = \pm, \quad j = 1, \ldots, n$. Then

$$(C_1, \ldots, C_n) \in \overline{T}_f\,(\theta_1, \ldots, \theta_n)\,,$$

where the constants $C_1, \ldots, C_n$ determine from the following formulas:

$$C_j \sin(2\alpha_j) = A_j^+ \sin(\theta_j + \alpha_j) + A_j^- \sin(\alpha_j - \theta_j)\,, \;\; j = 1, ..., n.$$

Remark Theorem is sharp: that is, there exists a function f for whom: the assumptions of theorem are satisfied, and the inequality is an equality.

This is a joint work with Armen Vagharshakyan.

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The Self-Interacting Dirac fields in FLRW spacetime

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In this talk we study solutions of the semilinear Dirac equation in the curved spacetime of the FLRW models of cosmology. We describe the relationship between the mass term, scale factor, nonlinear term, and initial function, which provides a global in time existence or an estimate on the lifespan of the solution of the Dirac equation in the expanding universe. The conditions on the imaginary part of mass will be discussed by proving nonexistence of the global solutions if certain relation between scale factor and the mass are fulfilled.

Phase transitions in logic

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We will cover several attractive examples of phase transitions related to unprovability results for first order Peano arithmetic.

Here the underlying idea is to consider parameterized sentences which are provable in arithmetic for small parameters and which become unprovable in arithmetic for large enough parameters.

Moreover we assume that if such a sentence is unprovable for some parameter then it stays unprovable for larger parameters as well.

It is then natural to investigate in these situation phase transition thresholds and phase transition windows around the threshold points.

In our presentation we will treat examples where the description of the thresholds is based on tools from mathematical analysis, like generating functions and Tauberian theorems.

Our results bridge a gap between mathematical disciplines which are unrelated at first sight. The talk will be addressed at a general mathematical audience and no specific familiarity with logic will be presupposed.

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Normal solvability and Fredholm properties of regular hypoelliptic operators

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We study normal solvability and Fredholm properties of Hörmander's hypoelliptic operators associated with multi-quasi-elliptic symbols. The considered class, termed regular hypoelliptic operators, is a generalization of elliptic, parabolic, $2b$–parabolic, and quasielliptic operators. The analysis of these operators has certain challenges, as their corresponding characteristic polynomials are "multi-quasi-elliptic" rather than homogeneous like in the elliptic case.

Solvability conditions, a priori estimates, and Fredholm properties have been studied for special classes of hypoelliptic operators in various functional spaces, but most of the results are related to elliptic and quasielliptic operators (see, for example, [1, 2, 3]). The Fredholm solvability for specific classes of regular hypoelliptic operators is studied in works [4, 5].

In this work, we establish normal solvability and a priori estimates for regular hypoelliptic operators with special variable coefficients, acting in multianisotropic Sobolev spaces. We obtain Fredholm criteria for the special classes of regular hypoelliptic operators in various scales of weighted and non-weighted multianisotropic spaces. We also provide applications to the smoothness of solutions, index invariance on the scale, and spectral properties of such operators.

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On polynomial solutions of PDE with constant coefficients

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Let Π_n^k be the space of polynomials in k variables of total degree at most n and let Π^k be the space of all polynomials in k variables.

Suppose that for a polynomial $p \in \Pi^k$ the point $\theta := (0, \ldots, 0) \in \mathbb{C}^k$ is an n_0 fold zero, i.e. p does not contain any monomial of total degree $\leq n_0 - 1$ and contains at least a monomial of total degree n_0.

In this paper, we prove that then for each $q \in \Pi_n^k$ the partial differential equation

$$p(D)f = q \tag{1}$$

has a polynomial solution from the space $\Pi_{n_0+n}^k$.

Moreover, for any $m \geq 0$, the homogeneous PDE $p(D)f = 0$ has exactly

$$\binom{m+k}{k} - \binom{m-n_0+k}{k}$$

linearly independent solutions in Π_m^k.

Note that obviously the PDE (1) has no polynomial solution from the space $\Pi_{n_0+n-1}^k$ if $q \notin \Pi_{n-1}^k$.

Let us now consider the special case $n_0 = 0$, i.e. $p(\theta) \neq 0$. Then the homogeneous PDE has only a trivial solution. We therefore obtain that the PDE (1) has a unique solution in Π_n^k which is at the same time the unique solution in each $\Pi_m,\ m \geq n$.

At the end, a method for solving the PDE (1) is presented.

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Generalized Abel-Plana formula as a renormalization tool in quantum field theory

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In quantum field theory the vacuum expectation values of physical observables bilinear in the field operator diverge. Among the most important points in the investigations of those expectation values is the regularization of divergent expressions, separation of divergences and the renormalization. In problems with boundaries the boundary conditions imposed on the field operator modify the spectrum of vacuum fluctuations and as a consequence the expectation values of physical characteristics are shifted by an amount that depends on the bulk and boundary geometries and on the boundary conditions. For compact boundaries the eigenvalues of quantum numbers are quantized by the boundary conditions. Usually those eigenvalues are given implicitly, as roots of some transcendental equations. The expressions for vacuum expectation values of observables contain divergent series over the eigenvalues. In the analog problem without boundaries an integral appears instead of the series. As a result, the shifts in the expectation values of physical observables is expressed in the form of the difference of the divergent series and the corresponding integral. In problems with planar boundaries a finite integral representation for that difference is provided by the Abel-Plana summation formula. The generalization of the Abel-Plana formula suggested in [1] (see also [2]) allows to obtain similar representations for more general classes of series where the summation goes over the zeros of a given function. Applications have been discussed in quantum field theoretical problems with curved boundaries. In particular, they include problems with spherical and cylindrical boundaries in the Minkowski spacetime [3,4], where the eigenvalues of the radial quantum number are expressed in terms of the zeros of combinations of the Bessel function and its first derivative. Series over those zeros also appear in the vacuum expectation values for bilinear characteristics of quantum fields in braneworld models on background of anti-de Sitter spacetime [5]. We also discuss a summation formula for series over the zeros of the associated Legendre function of the first kind with respect to its order. Applications are given in problems with spherical boundaries in spacetime with constant negative curvature spatial sections and in de Sitter spacetime described by hyperbolic coordinates [6].

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On unitary representations of almost Abelian groups

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An almost Abelian group is a non-Abelian Lie group with a codimension-1 normal subgroup. These groups are significant in various fields of geometry and physics, making their comprehensive study highly valuable. This talk focuses on constructing irreducible unitary representations of almost Abelian groups using the Mackey machine, along with calculating the resulting stabilisers and orbits of the action of automorphisms on the dual space.

Joint work with Zhirayr Avetisyan and Gagik Melkumyan.

On weighted integral operators for solutions of $\overline{\partial}-$equation in the Siegel domain of C^n

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Let us write an arbitrary $\eta = (\eta_1, \eta_2, \ldots, \eta_n) \in C^n$ as $\eta = (\eta_1, \eta')$, where $\eta' = (\eta_2, \ldots, \eta_n) \in C^{n-1}$. Let $< \bullet, \bullet >$ be the Hermitean inner product in C^n, i.e.

$$< \omega, \eta >= \sum_{k=1}^{n} \omega_k \cdot \overline{\eta_k} \equiv \omega_1 \cdot \overline{\eta_1} + < \omega', \eta' >$$

for arbitrary $\omega = (\omega_1, \omega') \in C^n, \eta = (\eta_1, \eta') \in C^n$. Further, put

$$|\eta|^2 = \sum_{k=1}^{n} |\eta_k|^2 \equiv |\eta_1|^2 + |\eta'|^2, \quad \forall \eta = (\eta_1, \eta') \in C^n.$$

The Siegel domain in the space C^n is defined as follows:

$$\Omega_n = \{(\eta_1, \eta') \in C^n : Im\eta_1 > |\eta'|^2\}. \tag{1}$$

For $n = 1$, Ω_n can be naturally interpreted as the upper half-plane $\Pi_+ \subset C$. For complex number $\beta (Re\beta > -1)$ and for functions $f(\eta) \in C^1(\Omega_n)$, let us introduce two integral operators:

$$P_{n,\beta}(f)(\omega) = 2^{n-1+\beta} \cdot c_{n,\beta} \cdot \int_{\Omega_n} \frac{f(\eta)(Im\eta_1 - |\eta'|^2)^\beta}{(i(\overline{\eta_1} - \omega_1) - 2 < \omega', \eta' >)^{n+1+\beta}} dm(\eta), \quad \omega \in \Omega_n, \tag{2}$$

where $c_n(\beta) = \frac{1}{\pi^n} \cdot \frac{\Gamma(n+1+\beta)}{\Gamma(1+\beta)}$ and dm is the Lebesgue measure in $C^n \equiv R^{2n}$, and

$$T_{n,\beta}(\overline{\partial} f)(\omega) = 2^{n+\beta} \cdot c_{n,\beta} \cdot \int_{\Omega_n} < (\overline{\partial} f)(\eta), \eta - \omega > \frac{(Im\eta_1 - |\eta'|^2)^{\beta+1}}{(i(\overline{\eta_1} - \omega_1) - 2 < \omega', \eta' >)^{\beta+1}} \times$$
$$\times \sum_{p=0}^{n-1} C_{n-1}^p \frac{(-4)^p}{p+1+\beta} \left(\frac{(Im\omega_1 - |\omega'|^2)(Im\eta_1 - |\eta'|^2)}{|i(\overline{\eta_1} - \omega_1) - 2 < \omega', \eta' >|^2} \right)^p \times$$
$$\times \frac{(i(\overline{\omega_1} - \eta_1) - 2 < \eta', \omega' >)^{n-1}}{\left(|i(\overline{\eta_1} - \omega_1) - 2 < \omega', \eta' >|^2 - 4(Im\omega_1 - |\omega'|^2)(Im\eta_1 - |\eta'|^2) \right)^n} dm(\eta), \quad \omega \in \Omega_n, \tag{3}$$

where

$$< (\overline{\partial} f)(\eta), \eta - \omega >= \sum_{i=1}^{n} \frac{\partial f(\eta)}{\partial \overline{\eta_i}} (\overline{\eta}_i - \overline{\omega_i}).$$

In [1](**A.H.Karapetyan, 2006**) the integral representation

$$f(\omega) = P_{n,\beta}(f)(w) - T_{n,\beta}(\overline{\partial} f)(\omega), \quad \omega \in \Omega_n. \tag{4}$$

was established for functions $f(\eta) \in C^1(\Omega_n)$ satisfying together with $(\overline{\partial} f)(\eta)$ certain integral conditions. In the case of holomorphic functions the representation (4) was proved in [2](**F.Ricci, M.Taibleson,1983**), [3](**M.M.Djrbashian, A.E.Djrbashian,1985**)(when $n = 1$) and [4] (**M.M.Djrbashian, A.H.Karapetyan,198**
(when $n > 1$).

The following assertion is true:

Theorem. *Assume that* $u(\eta) = \sum_{k=1}^{n} u_k(\eta) d\overline{\eta_k} \quad (\eta \in \Omega_n)$ *is a* $\overline{\partial}-$*closed* C^1-*differential form of type* $(0,1)$ *with compact support. In other words,* $u(\eta) = (u_1(\eta), u_2(\eta), \ldots, u_n(\eta))$ *is a* C^1-*vector-function in* Ω_n *with compact support such that* $\partial u_k(\eta)/\partial\overline{\eta_j} \equiv \partial u_j(\eta)/\partial\overline{\eta_k} (1 \leq k, j \leq n), \eta \in \Omega_n$. *Then the function* $f_\beta(\omega) \equiv -T_{n,\beta}(u)(\omega)$ *is a solution of the equation* $\overline{\partial} f = u$.

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Discrete-time replicator equations and optimal transport networks

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Evolutionary game dynamics [1], including replicator dynamics, have a wide range of applications across various fields, such as biology and ecology, economics, social dynamics, multi-agent systems and learning. It rests upon dynamical systems methods applied to game theory, and can be described by many types of dynamical systems, including ordinary differential equations (e.g., the replicator equation), differential inclusions (the best response dynamics), reaction-diffusion systems. In this context, the inefficiency of Nash equilibrium with respect to the global system optimum in noncooperative games [2], which is quantified by the so-called *price of anarchy*, has received considerable attention.

In this talk, we consider an unexpectedly new field of application of replicator dynamics and present the discretized version of continuous time dynamics by proposing the discrete-time replicator equations for the study of optimal transport networks with congestion. We first introduce the concept of a *Wardrop optimal network* that admits *Wardrop optimal flows* that are both Nash equilibrium and system optimum, and are the only networks with the price of anarchy exactly equal to its least value 1, thus resolving the long-standing problem of Dafermos [3]. Then we present a novel dynamical model of optimal flow distribution on Wardrop optimal networks, using the ideas of evolutionary game theory, which unlike the classical game theory, focuses on the dynamics of strategy change. In fact, the primary way to study the evolutionary dynamics is through replicator equations that show the growth rate of the proportion of agents using a certain strategy and that rate is equal to the difference between the average payoff of that strategy and the average payoff of the agents population as a whole. The key idea is that replicators whose fitness is larger (smaller) than the average fitness of population will increase (decrease) in numbers. The dynamic stability analysis of stationary solutions of replicator equations complement the static approach to evolutionary games.

Our dynamical model is based on discrete-time mean field replicator equations given on probability simplices and generated by nonlinear similar-order preserving mappings [4, 5]. In particular, we consider the discrete-time replicator dynamical systems generated by convex differentiable functions [6], and by Schur potential functions [7], namely, by gradient vector fields of Schur-convex functions, which give rise to discrete-time replicator equations in partial derivatives; this will also be exemplified by employing complete symmetric functions, gamma functions, and symmetric gauge functions as those that generate the replicator dynamics. Our main focus is on analyzing the dynamics, convergence, and stability of the replicator dynamical system, providing insights into how these systems evolve over time. We discuss Nash equilibria, convergence to fixed points, and asymptotic stability conditions of the replicator equation dynamics that are studied with the use of the tools from dynamical systems theory such as Lyapunov functions. For the replicator dynamical system under study, the Nash equilibrium, the Wardrop equilibrium, and the system optimum represent the same point in the state space, i.e., flow in the network. We also discuss the results of simulation that validate the theoretical results, where, *inter alia*, we analyze the convergence rate of orbits of replicator dynamical system to fixed points for different types of functions that generate the replicator equation. The proposed replicator dynamical model can also be used in neural network analysis and learning dynamics [8].

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Isometry Theorems for Omega-Weighted Hardy Type Spaces

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The talk is devoted to some theorems on isometry and Paley-Wiener type theorems, interpolation and biorthogonal systems in several weighted Hardy type spaces of functions holomorphic in the unit disc, in the half-plane and in the entire complex plane, which is a part of the general theory of functions of omega-bounded type presented in [1].

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Control and stabilisation for the Burgers equation

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We give an overview of various results on exact and approximate controllability of the Burgers equations by an external force localised in space. It will be shown that, due to the strong nonlinear dissipation, some areas of the phase space are not accessible, even if the size of control and time are arbitrarily large. We next turn to the problem of controllability to trajectories and prove that ow generated by the problem in question is globally exponentially stable. Together with the classical result on local exact controllability due to Fursikov Imanuvilov, this proves that the Burgers equation is exactly controllable to trajectories in a nite time not depending on the initial states

On a Dirichlet Problem for one Class of Properly Elliptic Equation

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Let $D = \{z : |z| < 1\}$ be the unit disc of the complex plane and $\Gamma = \{z : |z| = 1\}$ its boundary. We consider in D an equation

$$\left(\frac{\partial}{\partial \bar{z}} - \mu \frac{\partial}{\partial z}\right)^N \left(\frac{\partial}{\partial z} - \nu \frac{\partial}{\partial \bar{z}}\right)^N U = 0, \quad (x, y) \in D, \tag{2}$$

where μ and ν are the complex constants, which satisfy the conditions: $\mu\nu \neq 0$,

$|\mu| < 1$, $|\nu| < 1$, that is the equation (2) is properly elliptic. We seek the solution U in the class of functions $2N$ times continuously differentiable in D and continuous in $D\bigcup\Gamma$ with derivatives up to the order $N-1$. This solution on the boundary Γ satisfies the Dirichlet condition:

$$\frac{\partial^k U}{\partial r^k}|_\Gamma = f_k(x,y), \qquad (x,y)\in\Gamma, \quad k=0,1,\ldots,N-1, \tag{3}$$

where $f_k \in C^{N-1-k}(\Gamma)$ are the given functions, and $\frac{\partial}{\partial r}$ and $\frac{\partial}{\partial\theta}$ are the derivatives by the modulus and argument of the complex number ($z = re^{i\theta}$) respectively.

It is known, that this problem is Fredholmian (see [1]). We want to find the defect numbers of the problem, that is the number of linearly independent solutions of homogeneous problem (when $f_k \equiv 0$) and the number of linearly independent conditions for the solvability of the in-homogeneous problem. If the solution U of the equation (2) belongs to the class $C^{2N}(D)\cap C^{(N-1,\alpha)}(D\cup\Gamma)$ and given boundary functions f_k for $(k=\overline{0,N-1})$ are in the class $C^{(N-k-1,\alpha)}(\Gamma)$, the problem was efficiently studied in numerous works (brief historical overview see in [2]). In particular, in [3] the formula for determination of the defect numbers of the problem (2), (3) by the coefficients of (2) was found.

In the talk we will present the new formula for the defect numbers of the problem, and investigate effectively the problem (2), (3) in the space of continuous functions, that is we will find in explicit form the linearly independent solutions of the homogeneous problem and the linearly independent conditions for solvability of the in-homogeneous problem.

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MindFlayer: Efficient Asynchronous Parallel SGD in the Presence of Heterogeneous and Random Worker Compute Times

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We study the problem of minimizing the expectation of smooth non-convex functions with the help of several parallel workers whose role is to compute stochastic gradients. In particular, we focus on the challenging situation where the worker compute times are arbitrarily heterogeneous and random. In the simpler regime characterized by arbitrarily heterogeneous but deterministic compute times, Tyurin and Richtárik [1] recently proposed the first optimal asynchronous **SGD** method, called **Rennala SGD**, in terms of a novel complexity notion called time complexity. The starting point of our work is the observation that **Rennala SGD** can have bad and even arbitrarily bad performance in the presence of random compute times. To advance our understanding of stochastic optimization in this challenging regime, we propose a new asynchronous **SGD** method, for which we coin the name **MindFlayer SGD**, and perform theoretical time complexity analysis thereof. Our theory and empirical results demonstrate the superiority of **MindFlayer SGD** over existing baselines, including **Rennala SGD**.

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On universal (in the sense of signs) Fourier series with respect to the Walsh system

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The talk is based on a joint works with Martin Grigoryan

In recent years, we have obtained several results (see [1-5]) related to the existence and description of the structure of functions (universal functions), whose Fourier series with respect to the Walsh system are universal in a certain sense for diferent function classes. Let $\Phi = \{\varphi_k(x)\}_{n=0}^{\infty}$ be an orthonormal system on $[0,1)$ and let $f \in \ L^1[0,1)$. We denote by $c_k(f)$ the Fourier coefcients of function f with respect to the system Φ, that is $c_k(f) = \int_0^1 f(x)\varphi_k(x)dx$.

Defnition 1. We say that a function$U \in \ L^1[0,1)$ is universal for the space $L^0[0,1)$ ($L^0[0,1)$ is the class of all almost everywhere finite, Lebesgue measurable functions on $[0,1)$) with respect to system Φ in the sense of signs, if for each function $f \in L^0[0,1)$ one can choose numbers $\delta_k = \pm 1$ so that the series $\sum_{k=1}^{\infty} \delta_k c_k(U)\varphi_k(x)$ converges to f almost everywhere on $[0,1)$ (almost everywhere or in measure on $[0,1)$).

Defnition 2. We say that a function $U \in \ L^1[0,1)$ is universal for a space $L^p[0,1)$ (a weighted space $L^p_\mu[0,1)$) with respect to system $\{\varphi_k(x)\}_{n=0}^{\infty}$ in the sense of signs, if for each function $f \in L^p[0,1)$ $(f \in L^p_\mu[0,1)), p > 0$ one can choose numbers $\delta_k = \pm 1$ so that the series $\sum_{k=1}^{\infty} \delta_k c_k(U)\varphi_k(x)$ converges to f in $L^p[0,1)$ metric,

The following theorems are true

Theorem 1. There is a function $U \in L^1[0,1)$ with strictly decreasing Fourier coefficients which is universal for all spaces $L^p[0,1)$ $p \in (0,1)$ with respect to the Walsh system in the sense of signs.

Theorem 2. For each $\epsilon > 0$ there exist a measurable set $E \subset [0,1]$, with measure $|E| > 1 - \epsilon$ and a weight

function $0 < \mu(x) \leq 1$ with condition $\mu(x) = 1$ on E, so that for every function $f \in L^1[0,1)$ one can fnd a function

$g \in L^p[0,1)$, which coincides with f on E and is universal for the weighted space $L^p_\mu[0,1)$ with respect to Walsh system in the sense of signs.

Note that there is no a function $U \in L^1[0,1)$ -universal for space $L^1[0,1)$ with respect to the Walsh system in the sense of signs. Note also that in [5] is proved that there is a function $U \in L^1[0,1)$ with strictly decreasing Fourier coefficients which is universal for all spaces $L^0[0,1)$ with respect to the Walsh system in the sense of signs.

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Irreversibility and Time's Arrow in a Classical Three-Body Dynamical System

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The question of a strict explanation of the quantum-classical transition for dynamical systems, the classical analogues of which are non-integrable chaotic Poincaré systems, remains open (see Arnold-Berry theorem [1]). To solve this problem, the answer to the question is fundamentally important, namely: is irreversibility fundamental to the description of classical Hamiltonian mechanics? To answer this question, we examined the classical three-body problem, formulating it in conformal Euclidean space and proving the equivalence of the developed representation to the Newtonian three-body problem [2]. It is shown that conformal Euclidean-Riemannian space with a local coordinate system makes it possible to discover new hidden symmetries of the internal motion of a dynamical system. This allows us to reduce the mathematical problem of three bodies instead of the 8th one to a 6th order system (six first order ODEs).

Nevertheless, the most important result of the developed representation is that the chronologizing parameter of the movement of a system of particles, which we call internal time, is, generally speaking, fundamentally irreversible. Moreover, in global Jacobi coordinates, using the example of a specific model of particle interaction, it is shown that internal time is three-dimensional, heterogeneous and chaotic. The last circumstance is equivalent to considering a 6th order system as a system of stochastic differential equations of Langevin type with a power of fluctuation equal to the Lyapunov exponent for thedivergence of two close geodesic trajectories. Using the distribution of geodesic trajectories in a current tube, the entropy and complexity criterion of a low-dimensional dynamical system are determined. Assuming that the initial randomness satisfies the Gauss-Markov correlation conditions, we obtain a second-order PDE for the probability distribution of the flow of geodesic trajectories in 6D phase space. By using the distribution of trajectories in a current tube, the entropy is constructed and a criterion for the complexity of a low-dimensional dynamical system is determined.

The connection between the geometry and topology of space and internal time is examined in detail, its features are analyzed - multidimensionality, heterogeneity and the arrow of time. A criterion for estimating the dimension of emerging stochastic fractal structures for the internal time of a three-body system is obtained.

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Beyond the Hodge Theorem: curl and asymmetric pseudodifferential projections

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We develop a new approach to the study of spectral asymmetry. Working with the operator curl $:= *\mathrm{d}$ on a connected oriented closed Riemannian 3-manifold, we construct, by means of microlocal analysis, the asymmetry operator — a scalar pseudodifferential operator of order -3. The latter is completely determined by the Riemannian manifold and its orientation, and encodes information about spectral asymmetry. The asymmetry operator generalises and contains the classical eta invariant traditionally associated with the asymmetry of the spectrum, which can be recovered by computing its regularised operator trace. Remarkably, the whole construction is direct and explicit, and does not involve analytic continuation or algebraic topology.

This is joint work with Matteo Capoferri (Heriot-Watt University).

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Maximal functions on graded Lie groups

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In this talk, we review criteria for the boundedness of lacunary and of full maximal functions on graded Lie groups.

Joint work with Michael Ruzhansky and Julio Delgado.

Almost everywhere convergence for Malmquist Takenaka series

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The Malmquist-Takenaka (MT) system is a complete orthonormal system in $H^2(\mathbf{T})$ generated by an arbitrary sequence of points a_n in the unit disk with $\sum_n(1-|a_n|) = \infty$. The point a_n is responsible for multiplying the nth and subsequent terms of the system by a Möbius transform taking a_n to 0. One can recover the classical trigonometric system, its perturbations or conformal transformations, as particular examples of the MT system. However, many interesting choices of the sequence a_n, the MT system is less understood. We prove almost everywhere convergence of the MT series for three different classes of generating sequences (a_n).

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On the divergence of subsequences of partial Walsh-Fourier sums

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A class of increasing sequences of natural numbers (n_k) is found for which there exists a function $f \in L[0,1)$ such that the subsequence of partial Walsh-Fourier sums $(S_{n_k}(f))$ diverges everywhere. A condition for the growth order of a function

$\varphi : [0, \infty) \to [0, \infty)$ is given fulfillment of which implies an existence of above type function f in the class $\varphi(L)[0, 1)$.

On the issues of modeling the elimination of deadlock situations and synchronization problems using Petri nets

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Petri nets are a modeling mechanism that allows you to describe both the possible states of a system and its possible actions. A Petri net consists of four elements: P - a set of positions, T - a set of transitions, I - an input function, and O - an output function. Positions describe the state, and transitions describe the actions taking place in the network. The network structure is a quartet of elements $C = (P, T, I, O)$, where P and T are finite sets of positions and transitions. The input function I maps a transition t to a set of input positions, $I(t_j)$, and the output function O maps a transition t to a set of output positions, $O(t_j)$, i.e., a transition can have both input and output positions, including repetitions.

In this talk, we study some issues of modeling synchronization problems using Petri nets. The features of synchronous and asynchronous models are shown. The problem of dining philosophers is modeled with the help of Petri nets. The constructed model incorporates the concept of priority, which helps to avoid possible deadlocks in the system. With the help of Colored Petri Nets, the well-known synchronization problem about smokers is modeled, revealing certain features of Colored Petri Nets in relation to actions and semaphores. Additionally, unlike the classical Petri Net, this network contains fewer transitions and positions, thereby reducing the required memory.

The talk is based on a joint work with Andrey Avetisyan.

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An analogue of the idea of Riemann surfaces in differential geometry and real analysis

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In short, the idea is to establish relationships between the surfaces M in R^3 and some complex functions $w(z)$ assocciated with M; accordingly the idea is to apply complex functions $w(z)$ in studying surfaces M. In particular, real functions $u(x, y)$ also constitute a surface; thus the idea also applies to real functions.

In some ways, this idea is similar to the idea of Riemann surfaces.

Application of this idea leads to some results in differential geometry. In particular, analogues of the main theorems of classical Nevanlinna theory (in complex analysis) are obtained that are valid for generalized minimal surfaces (in geometry).

We also pose some problems that, in my opinion, can lead to a new crossroads between complex analysis, differential geometry and real analysis.

On the convergence sets of operator sequences on spaces of homogeneous type

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Let $f = \{f_k(x), \quad k = 1, 2, \ldots\}$ be an infinite sequence of real functions. Denote by $C(f) \subset \mathbb{R}$ the convergence set of this sequence, i.e. the set of points $x \in \mathbb{R}$ such that $\lim f_n(x)$ exists. A classical theorem of Hahn-Sierpinski asserts that if the functions $\{f_n\}$ are continuous, then $C(f)$ is a $F_{\sigma\delta}$-set, and conversely, every $F_{\sigma\delta}$-set is a convergence set for a sequence of continuous functions.

Such characterization problems were commonly considered in many fields of analysis such as Fourier series, analytic functions on the unit ball, power series, differentiation theory and there are many published papers and open problems, some of which will be considered in the last section.

We obtain complete characterization theorems for certain operator sequences living in general spaces of homogeneous type. Those generalize the results of [1,2], where the operators were considered on the interval $[0, 1]$. By a quasi-distance on a set X we mean a non-negative function $d(x, y)$ defined on $X \times X$ such that

1. $d(x, y) = 0$ if and only if $x = y$,
2. $d(x, y) = d(y, x)$ for any $x, y \in X$,
3. $d(x, y) \le K(d(x, z) + d(z, y))$,

where $K > 0$ is a constant. Let μ be measure on a σ-algebra in X, containing all Borel sets and balls, such that

$$\mu(B(x, 2r)) \le C\mu(B(x, r)) < \infty.$$

Such a combination (X, d, μ) is called a space of homogeneous type.

Definition. *The distance of two sets A and B in a space of homogeneous type X will be denoted by*

$$\operatorname{dist}(A, B) = \inf_{x\in A,\, y\in B} d(x, y)$$

The notation $A \Subset B$ will stand for the relation $\operatorname{dist}(A, B^c) > 0$. *Note that $A \Subset B$ implies $B^c \Subset A^c$.*

Let $L^1(X)$ denote the space of Lebesgue integrable functions on a space of homogeneous type (X, d, μ) and suppose that $M(X)$ is the normed space of bounded measurable functions on X with the norm $\|f\|_M = \sup_{x\in X}|f(x)|$. We consider sequences of linear operators

$$U_n : L^1(X) \to M(X), \quad n = 1, 2, \dots,$$

which may have some of the following properties

P1) $\rho_n = \|U_n\|_{L^1\to M} < \infty$, $n = 1, 2, \dots$,

P2) $\varrho = \sup_n \|U_n\|_{L^\infty\to M} < \infty$,

P3) if $f \in L^1(X)$ is constant on an open set $G \subset X$, then $U_n(x, f)$ uniformly converges over any set $E \Subset G$.

P4) if $G \subset X$ is an open set with $\mu(G) < \infty$, then $U_n(x, \mathbb{I}_G) \to \mathbb{I}_G(x)$ almost everywhere as $n \to \infty$.

Theorem. *If an operator sequence U_n satisfies P1)-P4), then for any $G_{\delta\sigma}$ null-set $E \subset X$ and any $\varepsilon > 0$, there exists a function $f \in L^\infty(X)$ such that*

1) $\mu(\mathrm{supp} f) < \varepsilon$,

2) $U_n(x, f)$ *diverges at any* $x \in E$,

3) $U_n(x, f) \to f(x)$ *if* $x \in X \setminus E$.

Applications of this theorems in different context will be considered. Related paper [3] will also be discussed.

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Group Theoretical Principles and the use of the Banach Gelfand Triple

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Classical (Abstract) Harmonic Analysis is dealing with specific orthogonal expansions (Fourier series) over a specific locally compact Abelian group G. Engineers distinguish between continuous or discrete, periodic or non-periodic (meaning well decaying) functions, which is more or less the same. However, real signals are much more diverse, and thus methods of time-frequency analysis (localized Fourier transform), or Gabor Analysis (in its discretized form) has been developed in the last decades. The treatment of the natural problems arising there (convergence of double series consisting of non-orthogonal elements, in short frame theory) requires other function spaces than just the usual L^p-spaces. As it turned out the Segal algebra $S_0(G)$ (Feichtinger's algebra) and its dual $S_0^*(G)$, the space of mild distributions, provide the appropriate setting for many questions, even outside of time-frequency analysis. Together with the Hilbert space $L^2(G)$ they form a chain of spaces, with S_0 in L^2 in S_0^*, the so-called Banach Gelfand Triple. We will restrict our attention to $G = \mathbb{R}^d$ (Euclidean case).

Among others all three spaces are invariant under the (extended) Fourier transform, and there is a kernel theorem, allowing to identify the space of bounded linear operators from $S_0(\mathbb{R}^d)$ to $S_0^*(\mathbb{R}^d)$ with elements (so-called "kernels") in $S_0^*(\mathbb{R}^{2d})$. But there is also the description of such operators via the so-called spreading representation or alternatively (connected via the symplectic Fourier transform) the Kohn-Nirenberg symbol (both belonging to $S_0^*(\mathbb{R}^{2d})$ as well).

We will concentrate on group theoretical considerations which help to understand how to approximate the Fourier transform of a function in $S_0(\mathbb{R})$ with the help of FFT-based methods, or how one can verify that a given function $g \in S_0(\mathbb{R}^d)$ generates a (good) Gabor frame, i.e. a system that allows to write each $f \in L^2(\mathbb{R}^d)$ as an unconditionally convergent series of time-frequency shifted copies of the given Gabor atom g.

Delaunay triangulation in numerical solution of two-dimensional boundary value problems

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The popular method for numerical solution of some problems of mathematical physics is the finite elements method. This method needs a mesh of triangles. The convergence rate of iteration process of numerical solution of the problem by the finite elements method depends on geometrical configuration of the mesh.

We prove the following extremal property: Let M be a triangular mesh with set of knots K. Denote by $S(M)$ the sum of cotangents of interior angles of all triangles from M.

Theorem 1. *For any fixed set of knots K, the sum $S(M)$ as a function on mesh M reaches his minimum for Delaunay triangulation.*

Using this extremal property, the theorem is obtained, that for any fixed knots set, for numerical solution of Maxwell equation of magnetic field the optimal mesh is Delaunay triangulation.

Constraint Maps
(Overview & Recent Developments)

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I will discuss, at a heuristic level, constraint maps that minimize Dirichlet energy, potentially with a forcing term. Specifically, I will focus on their optimal smoothness and the resulting free boundary. Although the study of these maps began in the 1970s and saw further development in the 1980s, it was later neglected. Our recent research has revived this theory, offering deeper insights and uncovering intriguing connections between solution singularities and free boundaries.

This is based on joint works with A. Figalli (ETH), A. Guerra (ETH), S. Kim (Uppsala)

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On q^2-analogue of the one-dimensional non-homogeneous heat equation

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In this paper, we discuss the q^2-analogue of non-homogeneous heat equation in one-dimension. Namely, we consider non-homogeneous heat equation for Rubin's difference operator. Well-posedness results are presented in appropriate Sobolev type spaces. In particular, we show that the heat equation generated by Rubin's difference operator have unique solutions. We even show that these solutions can be represented by explicit formulas.

On typical and atypical asymptotic behavior of singular solutions to Emden–Fowler Type Equations

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Consider the equation

$$y^{(n)} = p(x, y, y', \dots, y^{(n-1)})|y|^k \operatorname{sign} y, \tag{4}$$

where $n \geq 2$, $k > 1$, p is a positive, continuous and Lipschitz continuous in the last n variables function. Consider also a special case of (4), namely

$$y^{(n)} = p_0 \left|y\right|^k \operatorname{sgn} y \tag{5}$$

with $p_0 > 0$.

We discuss the problem posed by I.Kiguradze (see [1, Problem 16.4]) on asymptotic behavior of all positive non-extensible (so-called "blow-up") solutions to this equation. It appears for $n = 2$ (see [1]), $n = 3, 4$ (see [2]), that if $p(x, y_1, y_2, \ldots, y_{n-1})$ tends to p_0 as $x \to x^* - 0, y_0 \to \infty, \cdots, y_{n-1} \to \infty$, then all such solutions have the power-law asymptotic behavior

$$y(x) = C(x^* - x)^{-\alpha}(1 + o(1)), \quad x \to x^* - 0, \tag{6}$$

with $\alpha = \frac{n}{k-1}$, $C = \left(\frac{\alpha(\alpha+1)\ldots(\alpha+n-1)}{p_0}\right)^{\frac{1}{k-1}}$.

The same is true for weakly super-linear equations.

Theorem 1. ([3]) *Suppose $p \in C(\mathbf{R}^{n+1}) \bigcap Lip_{y_0,\ldots,y_{n-1}}(\mathbf{R}^n)$ and $p \to p_0 > 0$ as $x \to x^*, y_0 \to \infty, \ldots, y_{n-1} \to \infty$. Then for any integer $n > 4$ there exists $K > 1$ such that for any real $k \in (1, K)$, any solution to equation (1) tending to $+\infty$ as $x \to x^* - 0$ has the power-law asymptotic behavior (3).*

In the case $n \geq 12$ even if we deal with equation (2), another type of asymptotic behavior of singular solutions appears (see [3, 4, 5]).

If we have more strong nonlinearity, then the power-law asymptotic behavior becomes atypical. The following theorem generalizes the results of [6]:

Theorem 2. *If $12 \leq n \leq 100000$, then there exists $k_n > 1$ such that at any point $x_0 \in \mathbb{R}$ the set of initial data of asymptotically power-law solutions to equation (2) has zero Lebesgue measure whenever $k > k_n$.*

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Fractional Fourier transform, harmonic oscillator propagators and Strichartz estimates

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Using the Bargmann transform, we give a proof of that harmonic oscillator propagators and Fractional Fourier Transforms (FFT) are essentially the same. We deduce continuity properties for such operators on modulation spaces, and apply the results to prove Strichartz estimates for the harmonic oscillator propagator when acting on modulation spaces. We also show that general forms of fractional harmonic oscillator propagators are continuous on suitable on so-called Pilipović spaces and their distribution spaces. Especially we show that FFT of any complex order can be defined, and that these transforms are continuous on strict Pilipović function and distribution spaces.

The talk is based on a joint work with Divyang Bhimani and Ramesh Manna.

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On harmonic Bergman type projections on mixed norm Besov spaces

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We study the boundedness of Bergman (type) projection operators T that continuously map weighted Lebesgue and some more general mixed norm spaces $L(p, q, \alpha)$ onto their harmonic subspaces $h(p, q, \alpha)$ for suitable parameters. A necessary and sufficient condition is found for the operators T to be bounded on mixed norm spaces $L(p, q, \alpha)$ over the unit ball in $\mathbb{R}^n$. To this end, we first define harmonic reproducing kernels P_α of Poisson–Bergman type given by a version of fractional derivative, and next prove sharp lower estimates for the kernels P_α and their mixed norms. Then we turn to mixed norm Besov spaces and define three-parameter Besov spaces $\Lambda^{p,q}_\alpha$ of smooth functions over the unit ball in $\mathbb{R}^n$. A new family of Bergman type operators is constructed whose members are true bounded projections from the Besov space $\Lambda^{p,q}_\alpha$ onto its harmonic subspace $h\Lambda^{p,q}_\alpha$.

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Integral transform approach to solving partial differential equations with variable coefficients

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In the talk, we present an integral transform approach to solving partial differential equations with variable coefficients such as generalized Tricomi, Euler-Poisson-Darboux equations, and Dirac and Klein-Gordon equations of quantum field theory in some curved space-times of cosmology. We briefly review some early results obtained by that approach, such as the fundamental solutions to the generalized Tricomi equation, the Klein-Gordon equation, the Dirac and generalized Dirac operators, the Huygens' and incomplete Huygens' principles, and the problem of the existence of global in time solutions for equations in curved space-times. Then we present the new results obtained by that approach for equations in space-times of cosmological models with black holes.

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Error identity for variational problems with linear differential operator and smooth tensor

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For an approximated solution of a variational problem (VP), the question arise: "How close to the exact solution it actually is?" To answer that, one can use so called functional a posteriori estimate. It's a powerful tool since it imposes no restrictions on the method of approximation. Theoretic results and explicit computation for the biharmonic VP can be found in paper [1]. For the class of monotonous operators, a form of the a posteriori identity was recently obtained in [2].

In gaining a posteriori estimates, the so called "error identity" (EI) plays a huge role. It describes the distance between the exact solution and an arbitrary function from the corresponding set. In the talk, we will discuss possible ways of getting the EI for problems with linear differential operators and additional smooth tensor. We will also provide a suitable form of the error identity. These results continue the study started in [3].

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On the Convergence of Negative-Order Cesaro Means of Fourier and Fourier-Walsh Series

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It is well known that the sequence of partial sums of the trigonometric series of a function contains a subsequence that converges almost everywhere. In this context, Menshov posed the following problem: does this assertion hold if we move from conventional convergence to Cesàro summation methods of negative order? In other words, will the sequence of Cesàro means of negative order of the Fourier series of an integrable function contain an almost everywhere convergent subsequence?

In [1], he constructed an example of an integrable function giving a negative answer to the above question. In [2], we proved that the *"bad"* functions can be chosen from $C[-\pi,\pi)$ ($C[-\pi,\pi)$- is the class of all functions continuous on $[-\pi,\pi)$).

For the trigonometric system, the following theorems are established:

Theorem 1 *There exists a function $g \in C[-\pi,\pi)$ such that for an arbitrary increasing sequence of natural numbers $\{m_k\}_{k=0}^{\infty}$ and for each $\alpha \in (-1,-\frac{1}{2})$, we have*

$$\operatorname{mes}\left\{x \in [-\pi,\pi] \mid \limsup_{k\to\infty} |\sigma_{m_k}^{\alpha}(x,g)| = +\infty\right\} > 0.$$

Theorem 2 *There exists a set $\Lambda = \{m_k\}_{k=0}^{\infty}$ of natural numbers with density $\rho(\Lambda) = 1$, such that for any positive number ε and for each $f \in L^0[-\pi,\pi)$, one can find a function $g \in C[-\pi,\pi]$ satisfying* $\operatorname{mes}\{x \in [0,1] \mid f(x) \neq g(x) < \varepsilon\}$ *such that for any $\alpha < 0$ (excluding $\alpha = -1,-2,-3,\ldots$), both the Cesàro means $\sigma_{m_k}^{\alpha}(x,g)$ and the partial sums $S_k(x,g)$ of the trigonometric Fourier series of function g converge uniformly on $[-\pi,\pi]$ as $k \to \infty$.*

In [3], we give an example of a Fourier-Walsh series of a continuous function for which any subsequence of (C,α) means, where $\alpha \in (-1,-\frac{1}{2})$, diverges on a set of positive measure. We investigate the possibility of obtaining convergence of a subsequence of negative order Cesàro means of Fourier series of summable functions in $L^\infty[0,1)$-norm, after modifying these functions on a set of arbitrarily small positive measure.

Moreover, the following theorem is obtained:

Theorem 3 *There exists an increasing sequence of positive integers $\Lambda = \{m_k\}_{k=0}^\infty$ of natural numbers with density $\rho(\Lambda) = 1$ and a function $U \in L^1[0,1)$ such that:*

a) *The coefficients of the function U with respect to the Walsh-Paley system $\{W_k(x)\}_{k=0}^\infty$ are positive and arranged in descending order.*

b) *For each function $f \in L^0[0,1)$ and for any $\varepsilon > 0$, one can find a function $g \in C[0,1)$ such that $\mathrm{mes}\{x \in [0,1) \mid f(x) \neq g(x) < \varepsilon\}$, and both the subsequence of Cesàro means $\sigma_{m_k}^\alpha(x,g)$ of order $\alpha < 0$, $\alpha \neq -1,-2,-3,\dots$, and the partial sums $S_k(x,g)$ of the Fourier series for the function g with respect to the Walsh system uniformly converge to it on $[0,1)$ as $k \to \infty$.*

c) $|\int_0^1 g(x)W_k(x)dx| = \int_0^1 U(x)W_k(x)dx$ *for* $k \in \mathrm{spec}(g)$.

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Construction of a Mathematical Model and Solution of the Optimization Problem for the Bending of a Beam of Variable Thickness

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In this work, we consider the bending problem of a beam of length l with variable thickness, subjected to a uniformly distributed load.

Using the modified theory of plate bending [1], we derive a mathematical description of the physical problem of beam bending.

Thus, a mathematical model for the plate bending problem is constructed, represented by a system of two differential equations with variable coefficients, with the unknown functions W and ϕ. Depending on the type of support at the ends of the plate, the boundary conditions are also determined.

The resulting system of differential equations has the following form:

$$\begin{cases} -8h(x)\frac{d\phi(x)}{dx} - 16\frac{dh(x)}{dx}\phi(x) = 12q, \\ E((h(x))^2\dfrac{d^3w(x)}{dx^3}) + (2Eh(x)\dfrac{dh(x)}{dx}\dfrac{d^2w(x)}{dx^2}) - (Ea_{55}(h(x))^2\dfrac{d^2\phi(x)}{dx^2}) - \\ -(2Ea_{55}h(x)\dfrac{dh(x)}{dx}\dfrac{d\phi(x)}{dx}) + 8\phi(x) = 0 \end{cases} \tag{1}$$

Performing a variable substitution, we transition to dimensionless quantities, where the dimensionless x coordinate ranges from 0 to 1.

We assume that there is an additional support in the middle of the beam, denoted by the dimensionless coordinate x_m which also belongs to the interval [0, 1]. The boundary conditions considered are:

$$\begin{cases} W(0) = 0, \quad \frac{d^2w(x)}{dx^2} = 0 \quad \text{at} \quad x = 0, \\ W(x_m) = 0, \quad \frac{d^2w(x)}{dx^2} = 0 \quad \text{at} \quad x = x_m, \\ W(1) = 0, \quad \frac{dw(x)}{dx} = 0 \quad \text{at} \quad x = 1, \end{cases} \tag{2}$$

To solve the system of differential equations, we used the method of collocation. We searched for the unknown functions

$$W(x) = \begin{cases} W_1(x), & x \in [0, x_m] \\ W_2(x), & x \in [x_m, 1] \end{cases}$$

and $\phi(x)$ in the form:

$$W = a_0 + \sum_{i=1}^{n} a_i x^i, \quad \phi = b_0 + \sum_{i=1}^{n} b_i x^i$$

where W and ϕ are polynomials, and the coefficients a_i and b_i are unknown [2]. Using the derived differential equations and boundary conditions, we created a system of linear equations with respect to the unknown coefficients. By solving this system, the functions $W(x)$ and $\phi(x)$ were determined.

Depending on the position of the additional support, i.e., the value of the dimensionless coordinate x_m, the maximum value of the function $W(x)$ was calculated for $W_1(x)$ in $[0,\ x_m]$ and $W_2(x)$ in $[x_m,\ 1]$. An analysis of the obtained numerical values was performed, and the optimal value of x_m was determined to minimize the expression $\min|\max(W_1(x)) - -\max(W_2(x))|$.

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On the universal property of double Fourier series

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In [1] we construct an integrable function of two variables for which the double Fourier-Walsh series converges both by rectangles and by spheres and it is proved that after a suitable choice of signs for the Fourier coefficients of the series the spherical partial sums of the obtained series are dense in $L^p[0,1)^2$, $p \in (0,1)$.

The following theorem is true:

Theorem 1. There exist numbers $\{\delta_{k,j} = \pm 1\}_{k,j=0}^{\infty}$ and a function $U \in L^1[0,1)^2$ with $\mathrm{supp}(U) \subset [0,\varepsilon]^2$ (here $\varepsilon \in (0,1)$ is a given number) such that the Fourier series of U by the double Walsh system converges to U both by rectangles and by spheres. Besides, the coefficients of the series on the spectrum are positive and are arranged in decreasing order in all directions, and the spherical and rectangular partial sums of the series

$$\sum_{k,j=0}^{\infty} \delta_{k,j} c_{k,j}(U) W_k(x) W_j(y)$$

are dense in $L^p[0,1)^2$, $p \in (0,1)$,

(here $c_{k,j}(U)$ are the Fourier-Walsh coefficients of function U that is, $c_{k,j}(U) = \int_0^1 \int_0^1 U(t,\tau) W_k(t) W_j(\tau) dt d\tau$ $(k,j = 0,1,2,...)$.

Note that this theorem is in a sense final, since there is no integrable function of two variables for which the spherical and rectangular partial sums of the double Fourier-Walsh series would be dense in $L^p[0,1)^2$, $p \in (0,1)$.

Note also that S.V. Konyagin answering (see [**2**]) a question raised by Martin Grigoryan proved:

Theorem 2. (Konyagin). There exists no function $U \in L^1[-\pi,\pi)^d$, $d > 1$, whose Fourier series in the d-dimensional trigonometric system is universal over rectangles in $L^p[-\pi,\pi)^d$, $p \in (0,1)$. At the same time, the following theorem holds:

Theorem 3. (Grigoryan [3]). For any $p \in (0,1)$, $d \geq 2$, there exists a function $U \in L^1[-\pi,\pi)^d$, that is almost universal over rectangles and spheres alike for the class $L^p[-\pi,\pi)^d$, $p \in (0,1)$, with respect to the d-dimensional trigonometric system.

It turns out that each measurable function which is finite almost everywhere can be transformed into an almost universal function by changing its values on a set of arbitrarily small measure (see [**4**]).

Note that the Theorems 1-3 are true for the d-dimensional Walsh system.

This is a joint work with Martin Grigoryan.

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On Large Deviation Principle for Tapered Quadratic Functionals of Stationary Gaussian Processes

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In this talk we discuss the large deviation principle (LDP) for some important time series statistics. We present sufficient conditions that ensure LDP for tapered Toeplitz type quadratic functionals and forms of discrete and continuous time stationary Gaussian processes, as well as for the log-likelihood ratio of two stationary Gaussian distributions.

Wavelet Frames on the Space of M-positive Vectors

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Wavelets on the sets of M-positive vectors in the Euclidean space are studied. These sets are multidimentional analogs of the half-line in the Walsh analysis. Following the ideas of the Walsh analysis, the space of M-positive vectors is equipped with a coordinate-wise addition. Harmonic analysis on this space is also similar to the Walsh harmonic analysis, and the Fourier transform is such that there exists a class of so-called test functions (with a compact support of the function itself and of its Fourier transform). Tight wavelet frames consisting of the test functions are studied. A complete description of the masks generating such frames is given, and an algorithmic method for constructing them is developed. These frames may be useful for applications to signal processing because some examples of such systems on the half-line were already investigated in this aspect, and it appeared that they have an advantage over classical wavelet systems when used for processing fractal signals and images.

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Lipschitz regularity of a weakly coupled vectorial almost-minimizers for the p-Laplacian

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This is a joint work with Morteza Fotouhi and Henrik Shahgholian; [1]. For a given constant $\lambda > 0$, any $1 < p < \infty$, and a bounded Lipschitz domain $D \subset \mathbb{R}^n$ $(n \geq 2)$, we establish that almost-minimizers of the functional

$$J(\mathbf{v}; D) = \int_D \sum_{i=1}^{m} |\nabla v_i(x)|^p + \lambda \chi_{\{|\mathbf{v}|>0\}}(x)\, dx, \tag{7}$$

where $\mathbf{v} = (v_1, \cdots, v_m)$, and $m \in \mathbb{N}$, exhibit optimal Lipschitz continuity in compact sets of D. Furthermore, assuming $p \geq 2$ and employing a distinctly different methodology, we tackle the issue of boundary Lipschitz regularity for $\mathbf{v}$. This approach simultaneously yields alternative proof for the optimal local Lipschitz regularity for the interior case.

Our objective is to expand upon the findings presented in the recently published work [2], which addresses the local Lipschitz regularity of almost-minimizers in the context of Equation (7). Specifically, we aim to extend these results to cover any value of p within the range of $1 < p < \infty$ in a vectorial setting. It's worth noting that the outcomes reported in [2] were limited to the parameter range where $p > \max\left\{\frac{2n}{n+2}, 1\right\}$ and $m = 1$. Our approach for establishing the Lipschitz continuity of almost-minimizers will differ from the methodology employed in [2].

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Very weak solutions for PDEs

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In this talk we will give a review of the notion of very weak solutions and its applications, mostly for evolution equations with singular time-dependent coefficients.

A framework on generalised Fock spaces

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We consider pairs of weighted shift operators on a weighted l^2 space with weights associated with an entire function. When considered as operators over a general Fock space the commutators of these pairs of weighted shift operators are diagonal operators. We establish a calculus for the algebra of these commutators. As examples, we present the general case of Gelfond-Leontiev derivatives. This construction allows us to establish a general framework, which goes beyond the classic Weyl-Heisenberg algebra.

Inversion of the two data spherical radon transform using local data

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Hyperplane is a set of non-injectivity of the spherical Radon transform (SRT) in the space of continuous functions in $\mathbb{R}^d$. The, problem arises: to find an additional condition that allows one to reconstruct an unknown function $f \in C(\mathbb{R}^3)$ (not necessarily with compact support), using the spherical Radon transform over spheres centered on a plane. It is proved that this problem is equivalent to the injectivity of the following so-called two data spherical Radon transform.
Given a continuous function $f \in C(\mathbb{R}^3)$ for $(P,t) \in \{z=0\} \times (0,\infty)$ we define

$$Tf(P,t) = \frac{1}{4\pi} \int_{\mathbb{S}^2} (1 + i\cos\theta) f(P + t\omega)\, d\omega, \tag{8}$$

here $(\theta, \varphi) = \omega \in \mathbb{S}^2$ is the spherical coordinates of ω ($\theta \in [0,\pi]$ is the polar angle measured from the z-axis) and we integrate with respect to the surface Lebesgue measure on $\mathbb{S}^2$.

An inversion formula of the transform that uses the local data of the spherical integrals to reconstruct the unknown function is found. Such inversions are the mathematical base of modern modalities of imaging, such as Thermo and photoacoustic tomography and radar imaging, and have theoretical significance in many areas of mathematics.

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On directionally differentiable selections of set-valued mappings with convex, closed graphs

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A basic result on the existence of continuous selections concerning the case of lower semi-continuous mappings with closed convex images is the Michael theorem. In the paper we prove theorems that are more precise than the Michael theorem for certain classes of set-valued mappings.

Let $a : R^n \to 2^{R^m}$ be a set-valued mapping. The graph of a is defined as follows

$$graph(a) = \{(x, y) \in R^n \times R^m : y \in a(x)\}.$$

Suppose $a : R^n \to 2^{R^m}$, then a selection for a is a single valued mapping $y : R^n \to R^m$ such that $y(x) \in a(x)\ \forall x$.

Theorem. Let $a : R^b \to 2^{R^m}$ be a set-valued mapping with convex, closed graph which is defined in a neighborhood $U(x_0)$ of a point $x_0 \in R^n$. Let $(x_0, y_0) \in graph(a)$. Then there are a constant $L > 0$ and directionally differentiable selection y of a, defined in some neighborhood $V(x_0) \subset U(x_0)$, such that

$$y(x_0) = y_0,\ ||y(x_1) - y(x_2)|| \leq L||x_1 - x_2||^{1/2}\ \ \forall x_1, x_2 \in V(x_0).$$

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Transmission problems for a second order differential equation on a hypersurface with Lipschitz boundary in the generic Bessel potential spaces

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We are given a hypersurface $\mathcal{C} \subset \mathbb{R}^3$ with the Lipschitz boundary $\Gamma := \partial\mathcal{C}$, containing angular points $c_1, \ldots, c_n$. The surface is divided by a finite number of curves $\mathcal{T}_1, \ldots, \mathcal{T}_m$ in non-intersecting domains $\mathcal{C}_1, \ldots, \mathcal{C}_{m+1}$ and in each domain $\mathcal{C}_k$ is given Laplace-Beltrami equation with lower order perturbations $\Delta_{\mathcal{C}} u + \mathbf{P}_k(\mathcal{D})u = f_k$, $k = 1, \ldots, m+1$. The Dirichlet, Neumann and mixed type BVPs are considered on the outer boundary Γ, while on curves $\mathcal{T}_1, \ldots, \mathcal{T}_m$ are prescribed transmission conditions. The BVP is treated in a non-classical setting, when solutions are sought in the generic Bessel potential spaces with exponential weights $\mathbb{GH}^s_p(\mathcal{C}, \rho)$, $s > 1/p$, $1 < p < \infty$, $\rho(t) := \prod\limits_{j=1}^{n} |t - c_j|^{\beta_j}$. First we get rid of transmission conditions and transmission curves and reduce problem to the Fredholm equivalent Boundary Integral Equation (BIE) on the boundary of the surface $\Gamma = \partial\mathcal{C}$. Second we apply the localization and reduce the obtained BIE to the investigation of the Model BIE corresponding to Dirichlet, Neumann and mixed BVPs for the Laplace equation in a planar angular domains $\Omega_{\alpha_j} \subset \mathbb{R}^2$, $j = 1, 2, \ldots, n$, associated to the angular points $c_1, c_2, \ldots, c_n$. Third is investigated the model BIE in the generic Bessel potential spaces with weight $\mathbb{GH}^s_p(\Omega_{\alpha_j}, t^{\beta_j})$. For this we reduce further the BIE to a Fredholm equivalent Mellin convolution integral equations in the generic Bessel potential spaces on a semi-infinite axes with weight $\mathbb{GH}^{s-1/p}_p(\mathbb{R}^+, t^{\beta_j})$. Explicit criteria of Fredholm property of the initial BVPs are obtained. In contrast to the same BVPs in the classical Bessel potential spaces $\mathbb{H}^s_p(\mathcal{C})$, the Fredholm property in the generic Bessel potential spaces $\mathbb{GH}^s_p(\mathcal{C}, \rho)$ with weight is independent of the smoothness parameter s. We also list explicit singularities of solutions to the mixed-transmission BVP in the neighbourhood of knots, where boundary has angular points or Dirichlet-Neumann boundary conditions collide.

The presentation is based on the joint work with Medea Tsaava.

On Behavior of Fourier Coefficients by trigonometric System

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The present talk is devoted to studying the behavior of Fourier coefficients with respect to the trigonometric system of integrable functions.

Several papers (see [1]-[5]) have been devoted to correction theorems in which the absolute values of non-zero Fourier coefficients (by Haar Walsh and Faber-Schauder systems) of the corrected function decrease monotonically.

In [1] M. Grigoryan posed the following question:

Question. Is it possible for any function $f \in L^1[0,1]$ and for any $\varepsilon > 0$ to find a function $g \in L^1[0,1]$ such that $\mu\{x \in [0,1] : f(x) \neq g(x)\} < \varepsilon$ and the sequence of coefficients (or magnitudes of coefficients) in the expansion of the function g in the trigonometric system would be monotonically decreasing?

In this direction, we obtained the following result:

Theorem. For any $0 < \varepsilon < 1$ and for each function $f \in L^1[-\pi, \pi]$, there exists a measurable set $E \subset [-\pi, \pi]$ with measure $|E| > 2\pi - \varepsilon$ such that one can find a function $g \in L^1[-\pi, \pi]$ which coincides with f on E, and the sequence of Fourier coefficients of the newly obtained function with respect to the trigonometric system is in decreasing order.

This is a joint work with Martin Grigoryan.

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Subsequences of sequences of multiple partial trigonometric Fourier sums

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Let $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$, $d \geq 2$, and μ_d be the normalized Lebesgue measure on $\mathbb{T}^d$. Let $p > 0$ and $L^p(\mathbb{T}^d)$ be the set of measurable functions $f : \mathbb{T}^d \to \mathbb{C}$ such that

$$\|f\|_p^p := \int_{\mathbb{T}^d} |f(x)|^p d\mu_d < \infty.$$

For $f \in L^p(\mathbb{T}^d)$ we consider that $\|f\|_p \geq 0$. For $p \geq 1$ $\|\cdot\|_p$ is a norm. Also, we write $L(\mathbb{T}^d) = L^1(\mathbb{T}^d)$.

We associate with any function $f \in L(\mathbb{T}^d)$ its trigonometric Fourier series

$$f \sim \sum_{\mathbf{k}\in\mathbb{Z}^d} \hat{f}(\mathbf{k}) \exp(i\mathbf{kx}),$$

where $\mathbf{k} = (k_1, \ldots, k_d) \in \mathbb{Z}^d$, $\mathbf{x} = (x_1, \ldots, x_d) \in \mathbb{T}^d$, $\mathbf{kx} = k_1x_1 + \cdots + k_dx_d$. For a bounded subset $A \subset \mathbb{R}^d$ we define partial Fourier sums of f with respect to A

$$S_a(f)(x) = \sum_{\mathbf{k}\in\mathbb{Z}^d\cap A} \hat{f}(\mathbf{k}) \exp(i\mathbf{kx}),$$

We say that a sequence $\{A_j\}_{j\geq 1}$ of subsets of $\mathbb{R}^d$ is absorbing if for any $\mathbf{k} \in \mathbb{Z}^d$ and any $j \geq j(\mathbf{k})$ we have $\mathbf{k} \in A_j$.

Theorem 1 *Let $\{A_j\}$ be an absorbing sequence of bounded convex subsets of $\mathbb{R}^d$. Then there are a function $f \in L(\mathbb{T}^d)$ and an increasing sequence $\{j_\nu\}_{\nu\geq 1}$ of positive integers such that*

$$\lim_{\nu\to\infty} |S_{A_{j_\nu}}(f)(x)| = \infty$$

almost everywhere.

Theorem 2 *Let $p \in (0, 1)$ and $\{A_j\}$ be a sequence of bounded convex subsets of $\mathbb{R}^2$. Then the following conditions are equivalent:*

1) *there is $r \in \mathbb{N}$ such that for any $j \in \mathbb{N}$ the set A_j is covered by r parallel lines;*

2) *there is $r \in \mathbb{N}$ such that for any $j \in \mathbb{N}$ the set A_j is covered by r lines;*

3) *there is $c > 0$ such that for any j there is a line l_j $|\mathbb{Z}^2 \cap l_j \cap A_j| \geq c|\mathbb{Z}^2 \cap A_j|$; such that $|\mathbb{Z}^2 \cap l_j \cap A_j| \geq c|\mathbb{Z}^2 \cap A_j|$;*

4) *there is a constant $C > 0$ such that for any j*

$$\left\| \sum_{\mathbf{k} \in \mathbb{Z}^2 \cap A_j} \exp(i\mathbf{kx}) \right\|_1 \leq C \log(|\mathbb{Z}^2 \cap A_j| + 1);$$

5) *there is a constant $C > 0$ such that for any $f \in L(\mathbb{T}^d)$ the inequality $\|S_{A_j}(f)\|_p \leq C\|f\|_1$ holds;*

6) *there is no function $f \in L(\mathbb{T}^2)$ and no increasing sequence $\{j_\nu\}_{\nu \geq 1}$ of positive integers such that*

$$\lim_{\nu \to \infty} |S_{A_{j_\nu}}(f)(x)| = \infty$$

almost everywhere.

For any $d \geq 2$, the conditions 1)–6) in Theorem 2 can be considered for sets $A_j \subset \mathbb{R}^d$ with $\mathbb{R}^2$ replaced by $\mathbb{R}^d$ and $\mathbb{Z}^2$ replaced by $\mathbb{Z}^d$. The implications 1) $\to$ 2) $\to$ 3), 2) $\to$ 4), and 5) $\to$ 6) are obvious. The implication 6) $\to$ 4) is known. We show that the implication 2) $\to$ 5) holds. However, for $d = 3$ the implications 2) $\to$ 1), 3) $\to$ 2), 4) $\to$ 2), and 4) $\to$ 6) do not hold. We do not know if the implications 4) $\to$ 3) and 6) $\to$ 2) always hold.

Free and based-free involutions

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By a *space* we mean a separable metric space X. An involution is a continuous map $f : X \to X$ such that $f \circ f = Id_X$. An involution is called *free* if it has no fixed points, i.e., $f(x) \neq x$, $\forall x \in X$. An involution is called *based-free* if it has a unique fixed-point.

The main goal of this talk is to provide a brief summary of recent results on free and based free involutions, published in the papers [2] and [3].

For Hilbert space ℓ_2 we denote by σ the standard based-free involution $\sigma : \ell_2 \to \ell_2$ given by the formula $\sigma(x) = -x$. Let B denote the unit ball in ℓ_2, that is, $B := \{x \in \ell_2 \mid \|x\| \leq 1\}$. We prove that for every space (X, τ) with a based-free involution, the equivariant maps $(X, \tau) \to (B, \sigma)$ separate points and closed sets in X. That is, given a closet set $A \subset X$ and a point $x \in X \setminus A$, there exists an equivariant map $f : (X, \tau) \to (B, \sigma)$ such that $f(x) \notin \overline{f(A)}$. Here, the equivariantness of f means that it commutes with the given involutions, i.e., $f(\tau(x)) = \sigma(f(x))$ for all $x \in X$.

This is applied to show that (ℓ_2, σ) is universal in the sense that for each (X, τ) there exists an equivariant topological embedding $(X, \tau) \hookrightarrow (\ell_2, \sigma)$. Similarly, we prove that the unit sphere $\mathbb{S} = \{x \in \ell_2 \mid \|x\| = 1\}$, endowed with the standard free involution $x \mapsto -x$, is universal for all spaces X with a free involution $\tau : X \to X$. Another universal space with a free involution is the punctured Hilbert cube $[-1, 1]^\infty \setminus \{0\}$ endowed with the natural free involution $x \mapsto -x$.

We also investigate equivariant free and based-free compactifications and will present two new characterizations of based-free compactifications. It turns out that $\mathbb{S}$ has no free compactification, while ℓ_2 has no based-free compactification. We will explain why the countable product of real lines $\mathbb{R}^\infty$, endowed with the standard involution $x \mapsto -x$, is not universal for all based-free spaces. At the same time we will show that $\mathbb{R}^\infty$ is universal for those based-free spaces which admit a based-free compactification.

This is a joint work with Jan van Mill (University of Amsterdam, The Nether-

lands) and James E. West (Cornell University, USA).

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On the uniformly greedy universal functions with respect to the generalized Walsh system

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Let $a \geq 2$ be a fixed integer, and let $\omega_a = e^{2\pi i/a}$. The generalized Walsh system of order a defined as follows: $\psi_0(x) = 1$, and for any natural number n, with $n = \sum_{k=1}^{s} \beta_k a^{m_k}$, where $m_1 > m_2 > ... > m_s$, $\quad 0 < \beta_k \leq a-1$, we put

$$\psi_n(x) = \prod_{k=1}^{s} (\varphi_{m_k}(x))^{\beta_k} \ ,$$

where

$$\varphi_0(x) = \omega_a^k \ \ for \ \ x \in \left[\frac{k}{a}, \frac{k+1}{a}\right), \quad k = 0, 1, ..., a-1,$$

and $\varphi_n(x+1) = \varphi_n(x) = \varphi_0(a^n x)$ for any $n \geq 1$.

We denote the generalized Walsh system of order a as Ψ_a. Note that Ψ_2 is the classical Walsh system, and the system Ψ_a is a special case of the Vilenski system.

For a given function $f \in L^p[0, 1)$, where $p \geq 1$, we denote the Fourier coefficients of f with respect to the system Ψ_a as $c_k(f)$. The spectrum of a function $f(x)$ (denoted as $spec(f)$) is the set of indices for which the coefficients $c_k(f)$ are nonzero, i.e., $spec(f) = \{k \in N, c_k(f) \neq 0\}$.

Definition. The m-th greedy approximant of an element $f \in L^p[0,1)$, $p \geq 1$, with respect to the system Ψ_a is defined as the following sum:

$$G_m(f, \phi) = \sum_{k \in \Lambda} c_k(f)\phi_k, \tag{9}$$

where $\Lambda \subset 1, 2, \ldots$ is an arbitrary set of indices of cardinality m that satisfies the condition: $|c_n(f)| \geq |c_k(f)|$ if $n \in \Lambda$, $k \notin \Lambda$.

We say that the greedy algorithm for a function $f \in L^p[0,1]$, $p \geq 0$, converges with respect to Ψ_a if the sequence $G_m(x, f)$ converges to $f(t)$ in the L^p norm. Some results about the greedy algorithm for the generalized Walsh system can be found in [1] - [3].

The following statement holds:

Theorem. There exists a function $U \in L^1[0,1)$ with the following property: for any $0 < \epsilon < 1$ and any function $f \in \bigcap_{p>1} L^p[0,1)$, we can find a function $\tilde{f} \in \bigcap_{p>1} L^p$ such that $|x \in [0,1) : \tilde{f} \neq f| < \epsilon$, the greedy algorithm for the function $\tilde{f}$ with respect to the system Ψ_a, $a \geq 2$, converges uniformaly on $[0,1)$ and $|c_k(\tilde{f})| = c_k(U)$ for all $k \in spec(\tilde{f}).\geq 2$.

Similar issues in the $L^1[0,1)$ case are considered in [1].

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Some results in two-phase free boundary problem

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In this work we study minimizers of the energy functional

$$\int_\Omega |\nabla u|^2 + 2F(u)dx,$$

where $F'(u) \approx |u|^q \log u$ for some $-1 < q < 0$. Existence, optimal decay and non-degeneracy of solutions, from free boundary points are investigated. Consequently we derive the porosity property of the free boundary.

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From Harmonic Analysis to Machine Learning

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This talk explores the machine learning (ML) challenge of foreign accent conversion (FAC), a special speech processing problem, within the context of real-time applications. We provide a concise introduction to core ML concepts, highlighting its role in speech processing. A brief overview of digital signal processing (DSP) techniques for audio manipulation is also included.

We then delve into established FAC approaches, analyzing their strengths and weaknesses, particularly regarding their suitability for real-time scenarios. This analysis highlights the critical limitations of high parameter counts and extensive algorithmic lookahead that impede practical implementation.

We propose a novel FAC solution specifically designed to address these limitations, enabling real-time operation. Our approach prioritizes efficient parameter usage and restricted algorithmic lookahead, making it suitable for resource-constrained environments.

Finally, we establish objective evaluation metrics for FAC tasks and present the performance results achieved by our model.

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An uncertainty principle for operators

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We consider an irreducilble unitary representation ρ_λ of the twisted Heisenberg group $\mathbb{H}^n_\lambda = \mathbb{C}^n \times \mathbb{C}^n \times \mathbb{R}$ realised on the twisted Fock space $\mathcal{F}_\lambda(\mathbb{C}^{2n})$. To each element $\varphi \in \mathcal{F}_\lambda(\mathbb{C}^{2n})$ we can define two types of operators: S^λ_φ, which commute with all $\rho_\lambda(a,b), (a,b) \in \mathbb{R}^n$, and $\widetilde{S}^\lambda_\varphi$, which commute with all $\rho_\lambda(ia,ib), (a,b) \in \mathbb{R}^n$. Regarding the simultaneous boundedness of these operators on $\mathcal{F}_\lambda(\mathbb{C}^{2n})$ there is a very interesting uncertainty principle: for any non-constant $\varphi \in \mathcal{F}_\lambda(\mathbb{C}^{2n})$ at least one of these operators is unbounded. This is proved by using an analogue of Hardy's theorem for the Weyl transform.

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$C^{1,\alpha}$ Regularity For Stationary Mean-Field Games With Logarithmic Coupling

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The presentation is devoted to the investigation of stationary mean-field games (MFGs) on the torus with Lipschitz non-homogeneous diffusion and logarithmic-like couplings. The primary objective is to understand the existence of $C^{1,\alpha}$ solutions to address the research gap between low-regularity results for bounded and measurable diffusions and the smooth results modeled by the Laplacian.

We use the Hopf–Cole transformation to convert the MFG system into a scalar elliptic equation. Then, we apply Morrey space methods to establish the existence and regularity of solutions. The introduction of Morrey space methods offers a novel approach to address regularity issues in the context of MFGs.

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Grothendieck-Lidskii formulae in hypercomplex analysis

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The classic Grothendieck-Lidskii formula provides a connection between the trace and Fredholm determinant and the spectrum of a Fredholm operator. Unfortunately, linear algebra over non-commutative structures like Quaternions and Clifford algebras is quite different from classic linear algebra and, consequently, the classic formula does not hold in this case. In this talk we will discuss the difficulties and show how a type of Grothendieck-Lidskii formula can be established in these cases.

Approximate Controllability of PDEs Using Degenerate Forces

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In this talk, we will review some recent results on the controllability of PDEs using degenerate forces. In the case when the control force acts in the whole domain, we will see how approximate controllability can be established by applying large finite-dimensional controls on small time intervals through a carefully cho-

sen scaling. We will also discuss some results where PDEs are controlled using degenerate forces applied only in a small region of the domain.

This talk is partially based on joint works with Manuel Rissel.

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Orientation-Dependent Section Distributions for Convex Bodies

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Let $R^n (n \geq 2)$ be the n-dimensional Euclidean space, $\mathbf{D} \subset R^n$ be a bounded convex body with inner points and V_n be n-dimensional Lebesgue measure in R^n. The function $C(\mathbf{D}, h) = V_n(\mathbf{D} \cap (\mathbf{D} + h)), \quad h \in R^n$, is called the covariogram of $\mathbf{D}$. Here $\mathbf{D} + h = \{x + h, x \in \mathbf{D}\}$. G. Matheron conjectured that the covariogram of a convex body $\mathbf{D}$ determines $\mathbf{D}$ within the class of all convex bodies, up to translations and reflections (see [2] or [3]). Denote by S^{n-1} the $(n-1)$-dimensional sphere of radius 1 centered at the origin in R^n. We consider a random line which is parallel to $u \in S^{n-1}$ and intersects $\mathbf{D}$, that is a random line from the following set: $\Omega(u) = \{\text{lines which are parallel to } u \text{ and intersect } \mathbf{D}\}$. Let $\Pi r_{u^\perp}\mathbf{D}$ the orthogonal projection of $\mathbf{D}$ on the hyperplane $u^{\perp}$($u^{\perp}$ is the hyperplane with normal u and passing through the origin). A random line which is parallel to u and intersects $\mathbf{D}$ has an intersection point (denote by x) with $\Pi r_{u^{\perp}}\mathbf{D}$. We can identify the points of $\Pi r_{u^{\perp}}\mathbf{D}$ and the lines which intersect $\mathbf{D}$ and are parallel to u. The last means, that we can identify $\Omega(u)$ and $\Pi r_{u^{\perp}}\mathbf{D}$. Assuming that the intersection point x is uniformly distributed over the convex body $\Pi r_{u^{\perp}}\mathbf{D}$ we can define the following distribution function:

$$F(\mathbf{D}, u, t) = \frac{V_{n-1}\{x \in \Pi r_{u^{\perp}}\mathbf{D} : V_1(g(u,x) \cap \mathbf{D}) < t)\}}{b_{\mathbf{D}}(u)}.$$

The function $F(\mathbf{D}, u, t)$ is called orientation-dependent chord length distribution function of $\mathbf{D}$ in direction u at point $t \in R^1$, where $g(u, x)$ - is the line which is parallel to u and intersects $\Pi r_{u^{\perp}}\mathbf{D}$ at point x and $b_{\mathbf{D}}(u) = V_{n-1}(\Pi r_{u^{\perp}}\mathbf{D})$ (see [1] or [5]). We can introduce every vector $h \in R^n$ by $h = tu$, where u is the direction of h, and t is the length of h. Let $u \in S^{n-1}$ and $t > 0$ such that $\mathbf{D} \cap (\mathbf{D} + tu)$ contains inner points. Then $C(\mathbf{D}, u, t)$ is differentiable with respect to t and it holds that

$$-\frac{\partial C(\mathbf{D}, u, t)}{\partial t} = (1 - F(\mathbf{D}, u, t)) \cdot b_{\mathbf{D}}(u)$$

i.e. the problem of determining bounded convex domain by its covariogram is equivalent to that of determining it by its orientation dependent chord length distribution. In R^3 two types of orientation-dependent cross-section distributions can be considered. First is the probability that the random chord generated by intersection of the spatial line with the domain has length less than or equal to given number. In the second case random planes and their intersections with the domain are observed. The main goal is to enlarge the class of domains for which the form of the orientation dependent chord length distribution function and the cross-section area distribution function are known (see [4] or [6]).

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Stein-Weiss inequality on non-compact symmetric spaces

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In this talk, I will discuss the famous Stein-Weiss inequality, also known as the weighted Hardy-Littlewood-Sobolev inequality, in the framework of non-compact symmetric spaces of any rank. As an application, we will show that several weighted functional inequalities, such as Heisenberg's uncertainty principle, Gagliardo-Nirenberg's interpolation inequality, Pitt's inequality, etc., become available in this context. If time permits, we will also discuss some applications of these inequalities to partial differential equations.

This will be based on my joint works [1] and [2] with Aidyn Kassymov (Ghent University), Michael Ruzhansky (Ghent University), and Hong-Wei Zhang (Paderborn University).

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